3 Boats (A, B and C) are all together in the water. They then start moving with constant velocities. After 30 seconds, B is 100 meters @ 20º West of North away from A. The velocity of C with respect to B is 4.5 m/s @ 40º South of East. Where is boat C with respect to A at this time?

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Question is asking: $\mathbf{r}_{CA} = ?$

Key Ideas: $\mathbf{r} = \mathbf{v}t$ (because constant velocity.)

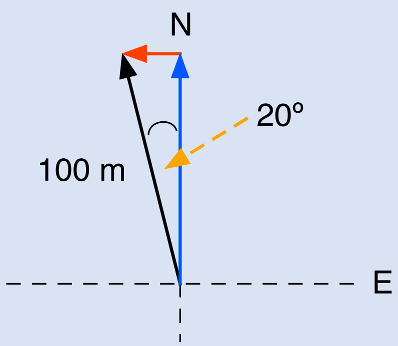
&

$\mathbf{r}_{CA} = \mathbf{r}_{CB} + \mathbf{r}_{BA}$ or $\mathbf{v}_{CA} = \mathbf{v}_{CB} + \mathbf{v}_{BA}$

3 Boats (A, B and C) are all together in the water. They then start moving with constant velocities. After 30 seconds, B is 100 meters @ 20º West of North away from A. The velocity of C with respect to B is 4.5 m/s @ 40º South of East. Where is boat C with respect to A at this time?

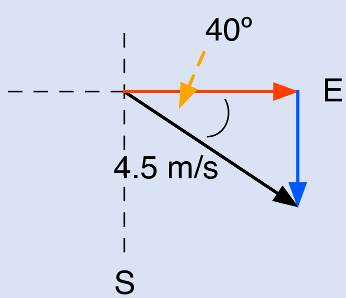
Solution 1: Solving with components (unit-vectors)

First, put the given information into unit vector form:



$\mathbf{r}_{BA}$ = –100sin(20) $\mathbf{i}$ + 100cos(20) $\mathbf{j}$ m
(or you could say $\mathbf{r}_{BA}$ = 100cos(110) $\mathbf{i}$ + 100sin(110) $\mathbf{j}$ m)

$\mathbf{r}_{BA}$ = –34.2$\mathbf{i}$ + 94$\mathbf{j}$ meters



$\mathbf{v}_{CB}$ = 4.5cos(40) $\mathbf{i}$ – 4.5sin(40) $\mathbf{j}$ m/s
(or you could say $\mathbf{v}_{CB}$ = 4.5cos(–40) $\mathbf{i}$ + 4.5sin(–40) $\mathbf{j}$ m/s)

$\mathbf{v}_{CB}$ = 3.45$\mathbf{i}$ – 2.89$\mathbf{j}$ m/s

Solution 1: Solving with components (unit-vectors)

Then decide if you want to add position or velocity vectors – and then add them.

$\mathbf{r}_{BA} = -34.2\mathbf{i} + 94\mathbf{j}$ m

$\mathbf{r} = \mathbf{v}t$, so

$\mathbf{r}_{CB} = \mathbf{v}_{CB}t = (3.45\mathbf{i} - 2.89\mathbf{j})(30)$

$\mathbf{r}_{CB} = 103.4\mathbf{i} - 86.8\mathbf{j}$ m

$$\mathbf{r}_{CA} = \mathbf{r}_{CB} + \mathbf{r}_{BA}$$
$$= (103.4\mathbf{i} - 86.8\mathbf{j}) + (-34.2\mathbf{i} + 94\mathbf{j})$$

$\mathbf{r}_{CA} = 69.2\mathbf{i} + 7.2\mathbf{j}$ m

$\mathbf{v}_{CB} = 3.45\mathbf{i} - 2.89\mathbf{j}$ m/s

$\mathbf{v} = \mathbf{r}/t$, so

$\mathbf{v}_{BA} = \mathbf{r}_{BA}/t = (-342\mathbf{i} + 94\mathbf{j})/30)$

$\mathbf{v}_{BA} = -1.14\mathbf{i} + 3.13\mathbf{j}$ m/s

$$\mathbf{v}_{CA} = \mathbf{v}_{CB} + \mathbf{v}_{BA}$$
$$= (3.45\mathbf{i} - 2.89\mathbf{j}\ ) + (-1.14\mathbf{i} + 3.13\mathbf{j})$$

$\mathbf{v}_{CA} = 2.31\mathbf{i} + 0.24\mathbf{j}$ m/s

$\mathbf{r}_{CA} = \mathbf{v}_{CA}t = (2.31\mathbf{i} + 0.24\mathbf{j})(30)$

$\mathbf{r}_{CA} = 69.2\mathbf{i} + 7.2\mathbf{j}$ m

3 Boats (A, B and C) are all together in the water. They then start moving with constant velocities. After 30 seconds, B is 100 meters @ 20º West of North away from A. The velocity of C with respect to B is 4.5 m/s @ 40º South of East. Where is boat C with respect to A at this time?

Solution 2: Solving with geometry

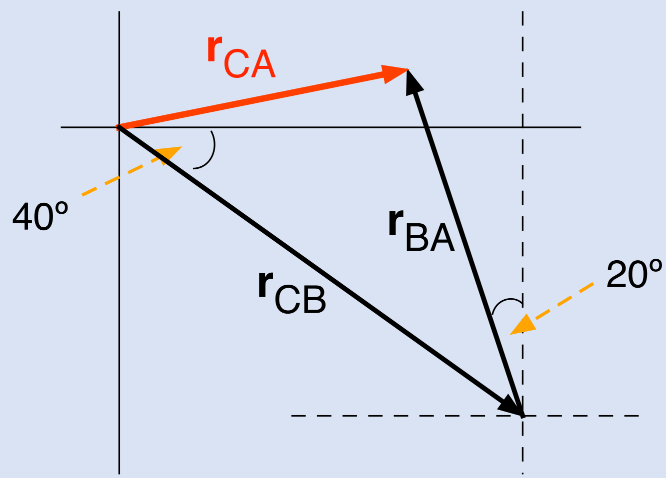
This means drawing the vector addition and making a triangle. Let's do the positions:

$$\mathbf{r}_{CA} = \mathbf{r}_{CB} + \mathbf{r}_{BA}$$

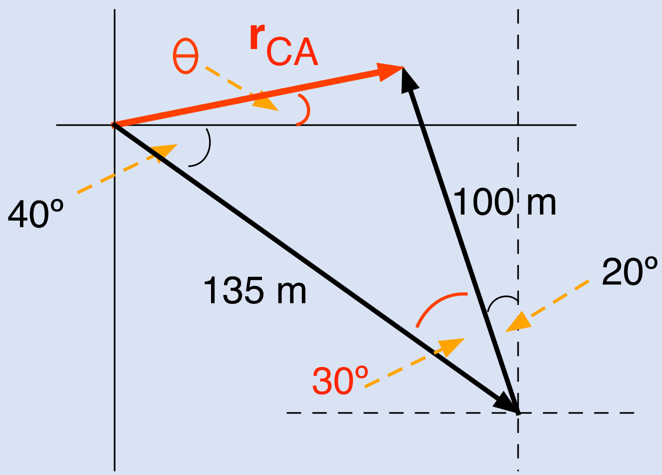
<u>$\mathbf{r}_{BA}$ = 100m @ 20º W of N</u>

$\mathbf{r}_{CB} = \mathbf{v}_{CB}t$ = (4.5)(30) @ 40º S of E

<u>$\mathbf{r}_{CB}$ = 135m @ 40º S of E</u>



Solution 2: Solving with geometry

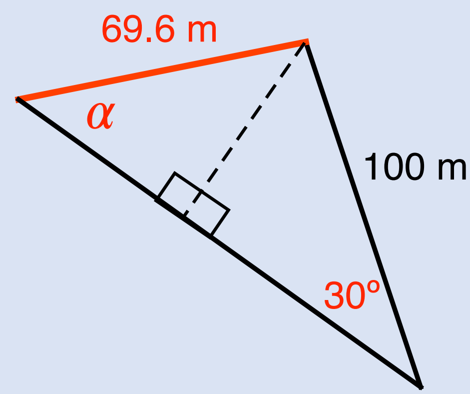


We have a triangle – so Law of Cosines gives

$$(\mathbf{r}_{CA})^2 = 135^2 + 100^2 - 2(135)(100)\cos(30)$$

$$r_{CA} = 69.6 \text{ m}$$

Drop an altitude (shown right) so we can say:

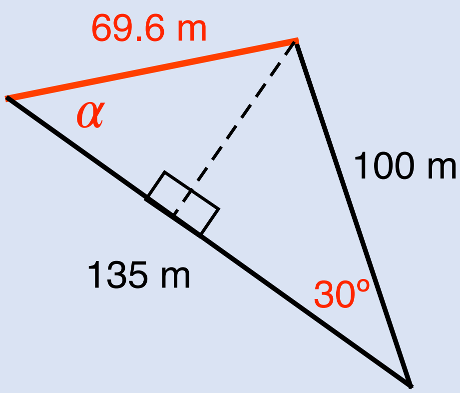


$$69.6\sin(\alpha) = 100\sin(30) \qquad \alpha = 45.9^\circ$$

Lastly, $\theta = \alpha - 40 = 45.9 - 40$

$$\theta = 5.9^\circ \text{ or } 5.9^\circ \text{ N of E}$$

Solution 2: Solving with geometry



Dropping an altitude is just the Law of Sines – but it makes clear what angle you are finding so you can avoid the ambiguous case finding an angle with the Law of Sines.

Alternately, use Law of Cosines again:

$$100^2 = 69.6^2 + 135^2 - 2(69.6)(135)\cos(\alpha)$$

$$\alpha = 45.9^\circ$$

3 Boats (A, B and C) are all together in the water. They then start moving with constant velocities. After 30 seconds, B is 100 meters @ 20º West of North away from A. The velocity of C with respect to B is 4.5 m/s @ 20º North of East. Where is boat C with respect to A at this time?

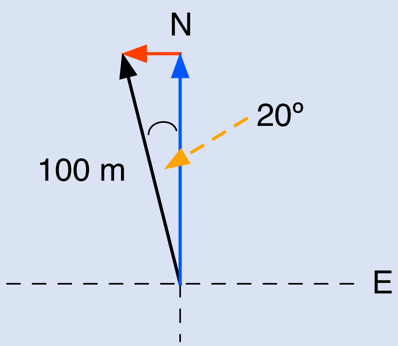
This is the same question, with just one direction changed (highlighted.)

Let’s see what happens.

3 Boats (A, B and C) are all together in the water. They then start moving with constant velocities. After 30 seconds, B is 100 meters @ 20º West of North away from A. The velocity of C with respect to B is 4.5 m/s @ 20º North of East. Where is boat C with respect to A at this time?

Solution 1: Solving with components (unit-vectors)

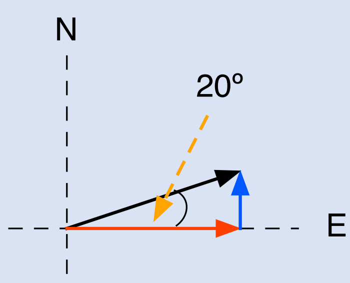
First, put the given information into unit vector form:



$$\mathbf{r}_{BA} = -100\sin(20)\,\mathbf{i} + 100\cos(20)\,\mathbf{j} \text{ m}$$

(or you could say $\mathbf{r}_{BA} = 100\cos(110)\,\mathbf{i} + 100\sin(110)\,\mathbf{j}$ m)

$$\mathbf{r}_{BA} = -34.2\mathbf{i} + 94\mathbf{j} \text{ meters}$$



$$\mathbf{v}_{CB} = 4.5\cos(20)\,\mathbf{i} + 4.5\sin(20)\,\mathbf{j} \text{ m/s}$$

$$\mathbf{v}_{CB} = 4.23\mathbf{i} + 1.54\mathbf{j} \text{ m/s}$$

Solution 1: Solving with components (unit-vectors)

Then add position or velocity vectors. Only the position vectors done this time.

$\mathbf{r}_{BA} = -34.2\mathbf{i} + 94\mathbf{j}$ m

$\mathbf{r} = \mathbf{v}t$, so

$\mathbf{r}_{CB} = \mathbf{v}_{CB}t = (4.23\mathbf{i} + 1.54\mathbf{j})(30)$

$\mathbf{r}_{CB} = 126.9\mathbf{i} + 46.2\mathbf{j}$ m

$\mathbf{r}_{CA} = \mathbf{r}_{CB} + \mathbf{r}_{BA}$

$= (126.9\mathbf{i} + 46.2\mathbf{j}) + (-34.2\mathbf{i} + 94\mathbf{j})$

$\mathbf{r}_{CA} = 92.7\mathbf{i} + 140.2\mathbf{j}$ m

Notice it is the exact same amount of work as the first version of the problem. The exact same process – just slightly different numbers.

3 Boats (A, B and C) are all together in the water. They then start moving with constant velocities. After 30 seconds, B is 100 meters @ 20º West of North away from A. The velocity of C with respect to B is 4.5 m/s @ 20º North of East. Where is boat C with respect to A at this time?

Solution 2: Solving with geometry

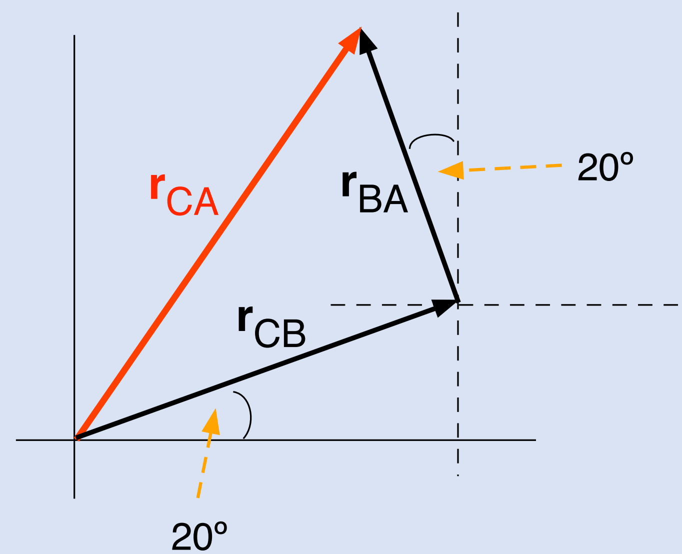
Again, make the position vectors and add them:

$$\mathbf{r}_{CA} = \mathbf{r}_{CB} + \mathbf{r}_{BA}$$

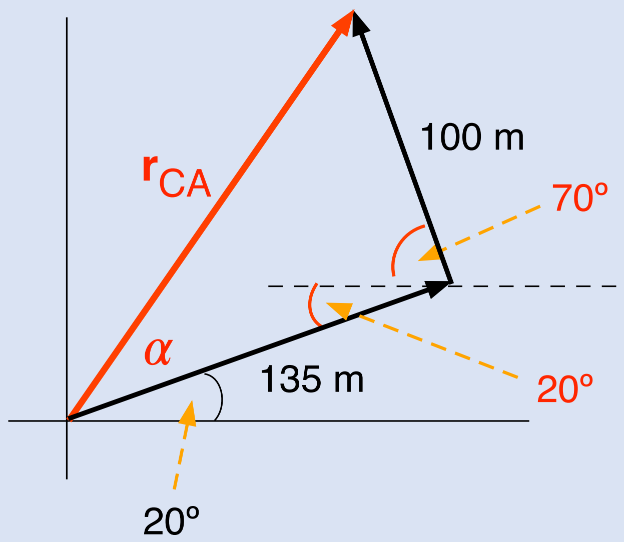
$\underline{\mathbf{r}_{BA} = 100\text{m} \ @ \ 20^\circ \text{ W of N}}$

$\mathbf{r}_{CB} = \mathbf{v}_{CB}t = (4.5)(30) \ @ \ 20^\circ \text{ N of E}$

$\underline{\mathbf{r}_{CB} = 135\text{m} \ @ \ 20^\circ \text{ N of E}}$



Solution 2: Solving with geometry



We have a right triangle!

$(\mathbf{r}_{CA})^2 = 135^2 + 100^2$ $r_{CA} = 168$ m

$\tan \alpha = 100/135$ $\alpha = 36.5º$

$\theta = \alpha + 20 = 36.5 + 20$ $\theta = 56.5º$

If you end up with a right triangle, the math part becomes pretty easy.